

1. The spectrophotometric approach to measure the color.

1.1 Measuring spectral data of the reflectance

$$(1.1.1) \quad R(\lambda) = \text{WhiteData}(\lambda) * M(\lambda) / W(\lambda)$$

where is:	$R(\lambda)$	reflectance spectrum
	$\text{WhiteData}(\lambda)$	reflectance spectrum of the white reference
	$M(\lambda)$	measured data on the sample
	$W(\lambda)$	measured data on the white reference

That means: The measurement of the reflectance spectrum is a relative measurement; just a comparison between the data measured on the white reference and on the sample.

1.2 Calculating the tristimulus values X,Y,Z (CIE 1931, CIE 1964)

$$(1.2.1) \quad \begin{aligned} X &= \sum S(\lambda) * x(\lambda) * R(\lambda) \\ Y &= \sum S(\lambda) * y(\lambda) * R(\lambda) \\ Z &= \sum S(\lambda) * z(\lambda) * R(\lambda) \end{aligned}$$

where is:	X,Y,Z	CIE 1931 or CIE 1964 tristimulus values
	$R(\lambda)$	reflectance spectrum
	$S(\lambda)$	spectrum of the illumination *)
	$x,y,z(\lambda)$	spectral tristimulus values **)

*) The tables of the illumination (daylight D65 for example) are defined by the CIE.

**) The spectral tristimulus values $x,y,z(\lambda)$ are representing the spectral sensitivity of the human eye. The tables are given in the CIE publications. For small samples (2°) there are the tables of the CIE 1931 publication, for large samples (10°) there are the tables of the CIE 1964 publication.

With the spectrophotometric approach, the calculations of the X,Y,Z are done numerically. With a colorimeter, these calculations are done by the hardware of the filters and of the light source. Changes of the characteristics of the spectrophotometer (light source, ...) do not change the results of X,Y,Z values. (see also 1.1).

1.3 Calculating the CIE L*a*b* values and CIE L*C*h* values (CIE 1964)

The (approximately) uniform color space Lab is a non linear transformation of the XYZ color space:

$$(1.3.1) \quad \begin{aligned} L^* &= 25 \cdot (100 \cdot Y/Y_0)^{1/3} - 16 \\ a^* &= 500 \cdot [(X/X_0)^{1/3} - (Y/Y_0)^{1/3}] \\ b^* &= 500 \cdot [(Y/Y_0)^{1/3} - (Z/Z_0)^{1/3}] \end{aligned}$$

where is:	X, Y, Z X ₀ , Y ₀ , Z ₀ L* a* b*	CIE 1931 or CIE 1964 tristimulus values X, Y, Z values of a perfect white sample CIE Lab L - value (lightness in Lab color space) CIE Lab a - value (red – green value) CIE Lab b - value (yellow – blue value)
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Color differences are calculated as follows:

$$(1.3.2) \quad \begin{aligned} \Delta L &= L_1 - L_2 \\ \Delta a &= a_1 - a_2 \\ \Delta b &= b_1 - b_2 \\ \Delta E &= (\Delta L^2 + \Delta a^2 + \Delta b^2)^{1/2} \end{aligned}$$

where is:	$\Delta L, \Delta a, \Delta b$ L ₁ , a ₁ , b ₁ L ₂ , a ₂ , b ₂ ΔE	Color differences in the CIE L*a*b* color space L*a*b* values of sample 1 L*a*b* values of sample 2 total color difference (distance between the 2 point in the color space)
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CIE LCh values: polar coordinates of the Lab values

$$(1.3.3) \quad \begin{aligned} L &: \text{as L in Lab} \\ C &= (a^2 + b^2)^{1/2} \\ h &= \arctan(b/a) \end{aligned}$$

where is:	L, a, b C h	L*a*b* values Chroma (saturation) in the L*a*b* color space Hue in the L*a*b* color space
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Differences in the CIE LCh color space:

$$(1.3.4) \quad \begin{aligned} \Delta L &= L_1 - L_2 \\ \Delta C &= C_1 - C_2 \\ \Delta E &= (\Delta L^2 + \Delta a^2 + \Delta b^2)^{1/2} \\ \Delta H &= (\Delta E^2 - \Delta L^2 - \Delta C^2)^{1/2} \end{aligned}$$

where is:	$\Delta L, \Delta C, \Delta H$ L ₁ , C ₁ , L ₂ , C ₂ $\Delta L, \Delta a, \Delta b, \Delta E$	Color differences in the CIE L*C*h* color space L*C* values of two samples see above
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2. Calculating the color data out of RGB data from a color sensor

2.1 Formula

$$(2.1.1) \quad (X,Y,Z) = A * (R,G,B)$$

$$(2.1.2) \quad (R,G,B) = A^{-1} * (X,Y,Z)$$

where is: X,Y,Z tristimulus values
 R,G,B R,G,B (red, green, blue) values from a color sensor
 A 3x3 matrix for the conversion *)
 A⁻¹ inverted matrix A

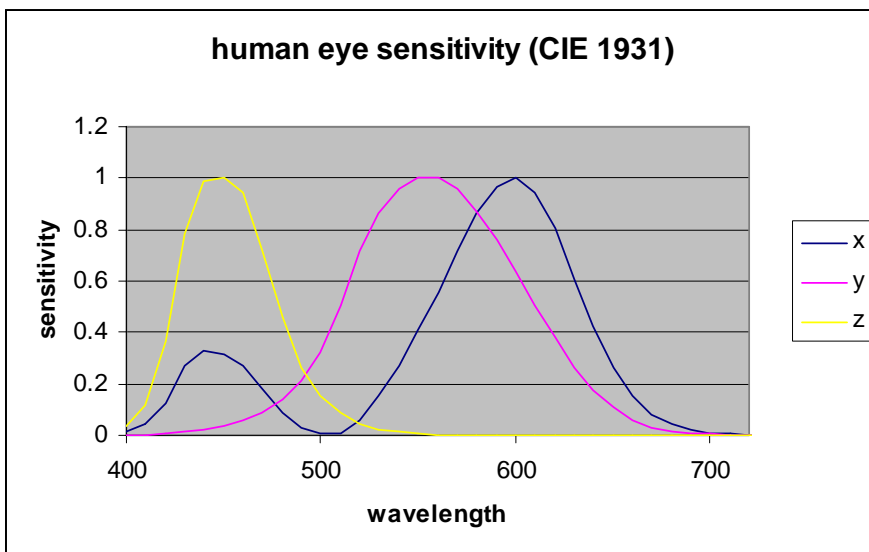
*) The 9 coefficients of the matrix are calculated to have the smallest error on the samples you are measuring. They depend on the filters and on the light source which are used in the measuring system (the camera).

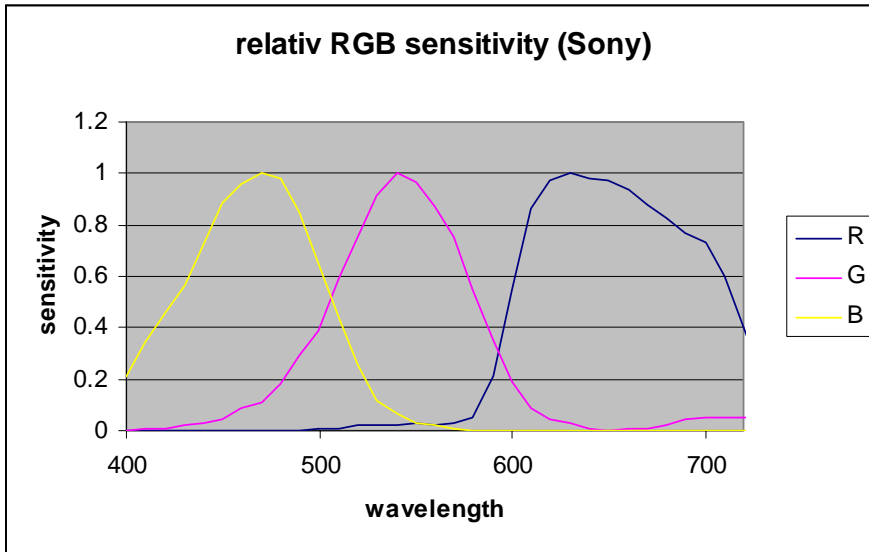
Based on the X,Y,Z values the Lab values can be calculated (see above).

2.1 Limitations of RGB based systems

The absolute precision, long term stability of those systems are weak. Because the filters of an RGB camera are not the same than the sensitivity of the human eye (spectral tristimulus values, x,y,z(λ)), the color cameras are some type of color blind (...).

The curves of the sensitivity are as follows:





That means the curves are quite different.

That means that such an RGB based color measurement gives wrong results: it is possible, that two different color spectrums look the same for the human eye, but the RGB based instruments detects a difference (and the other way round).

Such an instrument can only be used to compare very similar spectral curves.